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are divided by α . Hence the two substitutions which result from operating with the same number correspond in one of the possible simple isomorphisms between the given groups of multiplication and division, mod m .

In the more general case noted above, when a series of distinct natural numbers constitute a group as regards multiplication, we cannot always pass directly to a group of division without further restrictions. In fact, this more general set of numbers does not include unity with respect to the modulus, while the division of a number by itself gives unity for a quotient. It is, however, easy to see that whenever a set of numbers constitutes a group as regards multiplication, mod n , these numbers must also constitute a group as regards division, mod n , provided the quotient is restricted to the given set of numbers mod n .

As the converse of this proposition is evidently also true it results that *a necessary and sufficient condition that a set of distinct natural numbers constitutes a group with respect to division, mod m , when all of these numbers are divided successively by each one of them and the quotients are restricted to numbers of the set, is that m has the same highest common factor with each of these numbers and that the quotient obtained by dividing m by this factor is prime to this factor.* For instance, the set of numbers; 2, 4, 8, 10, 14, 16 constitutes the cyclic group of order 6 with respect to each of the operations of multiplication and division mod 18, provided that in the latter case the quotients are restricted to this set of numbers. In general, the groups of multiplication and division which are obtained in this manner are simply isomorphic, and the substitutions which correspond to the same number in these two groups are the inverses of each other.

CALIFORNIA TEACHERS OF MATHEMATICS.

The following extracts from the report of the last annual meeting of the Mathematics Section of the California High School Teachers' Association by the chairman, Professor Henry W. Stager, of Fresno Junior College, are significant in many ways, and will be of interest to readers of the MONTHLY:

At the annual meeting in July, 1914, the Mathematics Section of the California High School Teachers' Association adopted an official reading course for the present school year. The purpose of this course is to arouse a greater interest in the subject on the part of teachers of mathematics rather than to increase their knowledge of mere mechanical methods of presentation. Every teacher is urged to undertake the careful reading of one or more of these books this year. The course is divided into three sections, graded according to difficulty. Each teacher can find some book suited to his individual needs. The University of California will grant credit for the study of certain of the books as part of the work in university extension. Detailed information may be obtained by addressing the University Extension Division, University of California, and referring to the course herewith.

Teachers are urged to ask their trustees to place the entire list in the school

library at the earliest possible opportunity. The list is also suitable for public libraries,—in fact it forms a most excellent list for the general reader who is interested in mathematics and desires an introduction to the modern view-point. Teachers are asked to suggest the purchase of these books by the local public libraries. The list which follows has been recommended only after the most careful consideration by the committee in charge.

SECTION I. Books of easy grade.

1. Ball: *A Primer of the History of Mathematics.*
2. Cajori: *A History of Mathematics.*
3. Smith-Karpinski: *The Hindu-Arabic Numerals.*
4. Whitehead: *An Introduction to Mathematics.*

SECTION II. Books of medium grade.

5. Ball: *Mathematical Recreations and Essays.*
6. Beman and Smith: *Klein's Famous Problems in Elementary Geometry.*
7. J. W. Young: *Fundamental Concepts of Algebra and Geometry.*
8. Fine: *The Number-System of Algebra.*

SECTION III. Books of more advanced grade.

9. J. W. A. Young, Editor: *Monographs on Modern Mathematics.*

In connection with the above, the committee suggest the reading of a good mathematical journal and recommend for this purpose, THE AMERICAN MATHEMATICAL MONTHLY.

In his report, the chairman stated that he had sent out two letters to each of the 400 teachers of mathematics in the state, calling their attention to the study plan adopted by the Section in 1913. The work of the Section had also been presented at the four sectional meetings of the California Teachers' Association. As far as could be judged from correspondence and interviews with teachers, there seemed to be a new and increased interest in the better teaching of mathematics throughout the state. A message from Professor H. E. Slaught, managing editor of THE AMERICAN MATHEMATICAL MONTHLY, was read:

"Press on in the way you have started. Show the country what can be done by a body of teachers who have ambition to move *onward and upward*, who believe that the best enrichment of the secondary field depends upon the strongest and highest development of the teachers themselves, not simply through activity on the *dead level of daily routine*, but also, and most emphatically, through personal power gained in the higher ranges of thought and study. We are just beginning to realize the importance of this matter in this country, though other countries, and especially Germany, have given us object lessons in plenty. But this is part of the general uplift in our whole educational system and it will spread rapidly when it is once understood. In fact, this is the only explanation of the wonderful growth of the summer school idea in the universities. Thousands of teachers are resolving to take part in the uplift by *uplifting themselves through higher study and training*. To be in the lead in this great movement is no small honor. Hence, God speed the California teachers in this "forward look."

The chairman advocated a continuous policy in the work of the Section, especially for the coming year. To this end the previous committee on ways and means was reappointed, consisting of Professor D. N. Lehmer, chairman, Miss Thirmuthis Brookman, of San Mateo, and Miss Sadie L. Gilmore, of Colusa, together with the chairman of the Section.

Two of the papers read concerned the administration of the intermediate schools, sometimes called junior high schools, consisting of the seventh, eighth, and ninth grades.

Miss Thirumuthis Brookman, for some years head of the mathematics department of the Berkeley high schools, insisted that all mathematics must be correlated with the life of the students. She illustrated her principles by applications of the arithmetic of investment and expenditure, civic arithmetic, and the like, as worked out in Berkeley for the first two years of the intermediate school. The last year was devoted to algebra, which was made extremely practical, by the alternation of the work on the abstract principles of algebra with chapters dealing with the applications of proportion, such as belted pulleys in sewing machines, gears in mesh, levers, etc. The natural focus of the ninth-grade work was the mastery of the quadratic formula.

Mr. Will C. Wood, California commissioner of secondary schools, showed that mathematics should have a place in this curriculum, from both a *practical* and a *disciplinary* standpoint. Mathematics should be required throughout the three years, leading up to a mastery of arithmetic and the beginning of algebra in the eighth year. The ninth year should complete the essentials of elementary algebra and include some of the fundamental ideas of geometry, such as parallel lines, similarity, and simple geometric constructions, all woven together into a unified whole. The final topic which he considered was the preparation and certification of the teachers for this school. A special preparation will be necessary for this work, involving more time and study than that required for elementary school teachers, but not so highly specialized as the teacher of the secondary school. New methods of teaching should be effected in which the *individual* is the aim of the teacher's thought rather than *uniformity*.

Professor D. N. Lehmer, of the University of California, spoke with great effect on "How shall the isolated teacher of mathematics keep up his interest." He showed how mathematics, more than most other branches, could be a constant source of inspiration under any conditions, because neither laboratories nor immense libraries are essential in the search after her truths. He advised that teachers set apart a definite amount of time each week, even if it be only a few hours, and devote it to quiet and earnest consideration of some mathematical subject. After suggesting some topics for such study, he added that reviews of previous work are often exceedingly profitable. This address was full of inspiration and encouragement for every teacher.

Professor H. W. Marsh, of Pratt Institute, Brooklyn, gave a brief resumé of the educational ideals of the past and then showed how the modern ideal of educating each child so as to render him of greatest service to society is a transition in *name only* from the cultural ideal so long predominant. The main effect of the cultural plan has been to drive a very large percentage of our children from school by the end of the fifth grade. Many attempts have been made to introduce problems of real life into mathematics, but these problems have been largely of an artificial type. The solution of this question can only be brought about by pre-

senting the fundamental propositions and principles of mathematics in such constructive and developmental form as to *make each student feel the joy of individual discovery and creation*. The address contained some specific suggestions as to the subject matter and method of presentation of mathematics to meet these new ideals.

Professor Stager emphasized the need of greater coöperation between the university and the secondary school. This can be shown by a sympathetic interest on the part of the former in the varied, and often difficult, problems of the latter. High standards should be set by the university and should be insisted upon. Helpful suggestions should be offered through visitation of university instructors to the schools and teachers' meetings, and through carefully prepared bulletins. The university entrance requirements should receive re-adjustment: (1) by defining the two years of required mathematics as *elementary mathematics*, rather than as *algebra and plane geometry*,¹ which would permit of greater efficiency in teaching and afford those teachers favorable to the idea of so-called "unified mathematics" an opportunity to work out their ideas; (2) by allowing the second year of algebra to be devoted, one half to the more important, but less theoretical portions, of the present requirement, and the other half to the elementary fundamental principles of analytical geometry, more especially of the straight line and the circle. This would prove equally advantageous to both the university and the secondary school.

The two sessions were each attended by about seventy-five teachers and the papers were followed by spirited discussions.

BOOK REVIEWS.

EDITED BY W. H. BUSSEY, University of Minnesota.

Elementary Mathematical Analysis. A text-book for first year college students.

By CHARLES S. SLICHTER. McGraw-Hill Book Company, New York, 1914.
xiv + 490 pages.

Note.—The editors asked two men representing quite different phases of mathematical interests to review this book, and they take pleasure in presenting to the readers of the MONTHLY these two reviews.

I.

For years it has been assumed that mathematics should be administered in definite doses. After a pupil has emerged from the grammar school he is to be treated to one year of elementary algebra, one of plane geometry, then a half year each, of solid geometry, plane trigonometry, intermediate algebra, and advanced algebra. The tendency has been to prescribe the table of contents of each of these subjects. A student who can pass examinations in them all is ready to commence the study of analytic geometry, for which considerable familiarity

¹ This provision was made by the University of Chicago two years ago. EDITOR.